Final Exam - Review 2 - Problems

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1 Systems of differential equations

Problem 1

Solve $\mathbf{x}' = A\mathbf{x}$, and find the fundamental matrix $X(t)$, where:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 8 & -14 & 7 \end{bmatrix}$$

Problem 2

Solve $\mathbf{x}' = A\mathbf{x}$, where:

$$A = \begin{bmatrix} -1 & -2 & 0 \\ 8 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 3

Solve $\mathbf{x}' = A\mathbf{x}$, where:

$$A = \begin{bmatrix} 5 & -3 \\ 3 & -1 \end{bmatrix}$$

2 Grünbaum's coupled harmonic oscillator

Problem 4

Assume you're given a coupled mass-spring system with $N = 2$, $m_1 = m_2 = 1$, $k_1 = k_2 = k_3 = 1$. Find the proper frequencies and the proper modes of the system.

3 Partial differential equations and Fourier series

Problem 5

Find the Fourier cosine series of $f(x) = x^2$ on $(0, \pi)$

Problem 6

Find the solution of the following heat equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} = 4\dfrac{\partial^2 u}{\partial x^2} & 0 < x < \pi, \quad t > 0 \\ \dfrac{\partial u}{\partial x}(0,t) = \dfrac{\partial u}{\partial x}(\pi,t) = 0 & t > 0 \\ u(x,0) = x & 0 < x < \pi \end{cases} \tag{1}$$

Problem 7

Find the solution of the following wave equation:

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = 9\dfrac{\partial^2 u}{\partial x^2} & 0 < x < 1, \quad t > 0 \\ u(0,t) = u(1,t) = 0 & t > 0 \\ u(x,0) = 3\sin(3\pi x) & 0 < x < 1 \\ \dfrac{\partial u}{\partial t}(x,0) = 5\sin(4\pi x) & 0 < x < 1 \end{cases} \tag{2}$$

4 Higher-order differential equations

Problem 8

Are the functions xe^x, x^2e^x, x^3e^x linearly independent or dependent on $(-\infty, \infty)$?

Problem 9

Find the largest interval (a, b) on which the following differential equation has a unique solution:

$$(x-2)y'' + \ln(x)y' = \sqrt{3-x}$$

with $y(1) = 0, y'(1) = 2$.

Problem 10

(a) Solve $y''' - 3y'' + 3y' - y = 0$

(b) Find the form of a particular solution to $y''' - 3y'' + 3y' - y = e^t$

Problem 11

Solve $y'' + y = \tan(t)$ using variation of parameters.